

University of Colombo, Sri Lanka
University of Colombo School of Computing

Bachelor of Science in Computer Science

First Year Examination — Semester I (Sample Paper)

SCS 1306 — Linear Algebra

SAMPLE PAPER B

Two (2) Hours

Answer All Questions

Number of Pages = 16

Number of Questions = 4

To be completed by the candidate

Index Number

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Important Instructions to candidates

- Students should answer in the medium of English language only using the space provided in this question paper.
- Note that questions appear on both sides of the paper. If a page or a part of this question paper is not printed, please inform the supervisor immediately.
- Write your index number **CLEARLY** on each and every page of this Question paper.
- This paper consists 3 structured questions and 15 MCQ based questions on **16** pages.
- Answer **ALL** questions. Questions 1 and 4 carry 30 marks each, while Questions 2 and 3 carry 20 marks each.
- Do not tear off any part of this answer book. Under no circumstances may this book (or any part of this book), used or unused, be removed from the Examination Hall by a candidate.
- Calculators and any electronic device capable of storing and retrieving text including electronic dictionaries, smart watches and mobile phones are **not allowed**.

**To be completed
by the examiners**

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2	
3	
4	
Total	

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1.

- (a). Explain what is meant by a *pivot* in Gaussian elimination, and describe what must be done when a zero appears in a pivot position.

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- (b). Consider the following linear system.

$$x + 2y + z = 1$$

$$2x + 5y + 3z = 3$$

$$3x + 4y + 2z = 3$$

- i. Write the augmented matrix of the above linear system.

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- ii. Solve the above linear system using Gaussian Elimination with back substitution and find x , y and z .

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- (c). Explain Two (2) advantages of factorising a matrix as $A = LU$ rather than repeating Gaussian elimination each time the system is solved.

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- (d). Consider the following 3×3 matrix A .

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 8 & 5 \\ 3 & 13 & 12 \end{pmatrix}$$

- i. Find the lower triangular matrix L and the upper triangular matrix U such that $A = LU$.

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- ii. Using the factorisation obtained above, solve $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = (6, 15, 28)^T$. (*Do not re-run Gaussian elimination.*)

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2. Let $A = \begin{pmatrix} 4 & 1 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

(a). Determine the eigenvalues of the matrix A .

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(b). Find the linearly independent eigenvectors corresponding to each eigenvalue obtained in part (a).

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- (c). State, with reason, whether A is diagonalizable. If it is, determine an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

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- (d). Using the diagonalization obtained in part (c), determine A^3 .

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- (e). Calculate the determinant ($\det(A)$) and the trace ($\text{tr}(A)$) of the matrix A using the obtained eigenvalues.

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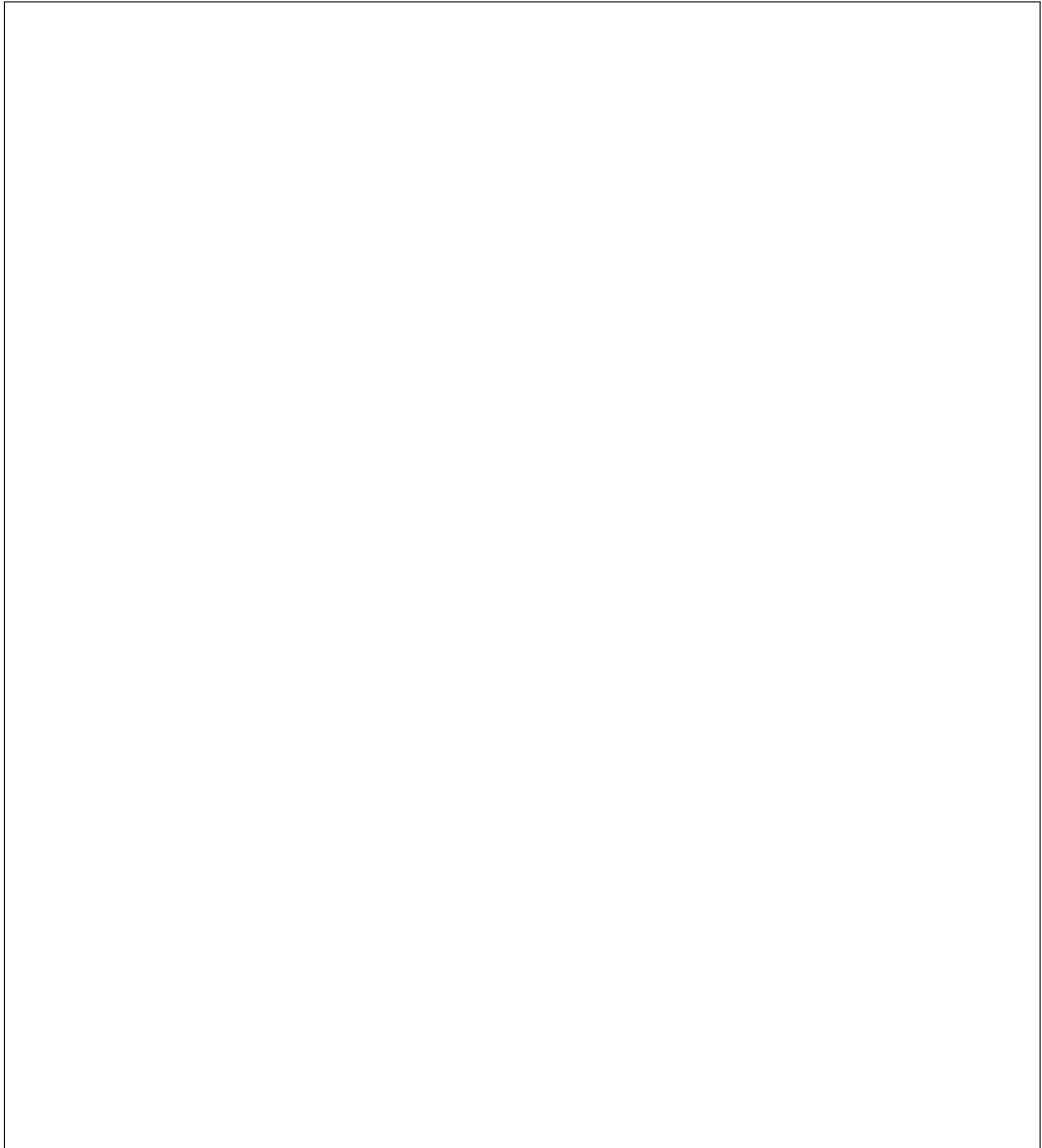
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3. The five corners of a house are given by the following column vectors.

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

- (a). Rotate the house 90° counterclockwise around the origin. Write down the rotation matrix used, draw both the original figure (A) and the rotated figure (B) on a cartesian coordinate plane (with x and y axes), and write down the resulting coordinates.



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4. Question 4 has 15 MCQ based questions. (2 marks x 15 = 30 marks)

- Each MCQ question will have 5 (five) choices with one correct answer.
- Mark the correct answers in the provided space at the last page of this paper.
- Cross or color the **best suite answer** among the given options.

(a). Which one of the following is NOT a linear equation?

$$3x + y - z = 5$$

$$x = 7$$

$$\sqrt{2}x - 4y = 0$$

$$xy + z = 4$$

$$x_1 + 2x_2 + 3x_3 = 0$$

(b). The inverse of the matrix $\begin{pmatrix} 4 & 7 \\ 2 & 6 \end{pmatrix}$ is

$$\frac{1}{10} \begin{pmatrix} 4 & -7 \\ -2 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 6 & -7 \\ -2 & 4 \end{pmatrix}$$

$$\frac{1}{10} \begin{pmatrix} 6 & -7 \\ -2 & 4 \end{pmatrix}$$

$$\frac{1}{10} \begin{pmatrix} 6 & 7 \\ 2 & 4 \end{pmatrix}$$

The inverse does not exist.

(c). Which of the following must be TRUE of an eigenvector of a matrix?

It must be the zero vector.

It must be a unit vector.

It must have integer entries.

It must be orthogonal to every other eigenvector.

It must be a non-zero vector.

(d). An $n \times n$ matrix A is diagonalizable if and only if

all of its entries are non-zero.

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it has n linearly independent eigenvectors.

it is symmetric, and only then.

all of its eigenvalues are distinct, and only then.

its determinant is non-zero.

(e). For an $n \times n$ matrix A and a scalar c , $\det(cA)$ equals

$c^n \det(A)$

$c \det(A)$

$nc \det(A)$

$\det(A)$

$c^{n-1} \det(A)$

(f). Cramer's rule may be used to solve $A\mathbf{x} = \mathbf{b}$ when

A is any rectangular matrix.

A is square and $\det(A) = 0$.

A is square and $\det(A) \neq 0$.

the system is homogeneous, and only then.

A is diagonal, and only then.

(g). Which one of the following matrices is symmetric?

$$\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix}$$

(h). For a matrix A of size $m \times n$, the column space $C(A)$ is a subspace of

$$\mathbb{R}^n$$

$$\mathbb{R}^{mn}$$

$$\mathbb{R}^{m+n}$$

$$\mathbb{R}^m$$

$$\mathbb{R}^2$$

(i). If a square matrix has determinant equal to zero, then

it has no eigenvalues.

it is singular and therefore has no inverse.

its rank is full.

it must be the zero matrix.

all of its entries are zero.

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(j). The rank–nullity theorem for a matrix A of size $m \times n$ states that

$$\text{rank}(A) + \dim N(A) = m$$

$$\text{rank}(A) \cdot \dim N(A) = n$$

$$\text{rank}(A) = \dim N(A)$$

$$\text{rank}(A) - \dim N(A) = n$$

$$\text{rank}(A) + \dim N(A) = n$$

(k). The determinant of the 2×2 rotation matrix $R_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ is

0

−1

$\cos \theta$

+1

$\sin \theta$

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(l). A transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ can be linear only if

$$T(\mathbf{0}) = \mathbf{0}.$$

T always increases the dimension of the space.

$$T(\mathbf{x}) = A\mathbf{x} + \mathbf{b} \text{ with } \mathbf{b} \neq \mathbf{0}.$$

T maps every vector to a unit vector.

T is invertible.

(m). Which one of the following correctly describes a basis of a vector space V ?

Any spanning set of V .

Any linearly independent set of vectors in V .

A set of vectors that is linearly independent and spans V .

The set of all vectors in V .

Any orthogonal set of vectors in V .

(n). The trace of the matrix $\begin{pmatrix} 3 & 1 & 4 \\ 0 & -2 & 5 \\ 7 & 8 & 1 \end{pmatrix}$ is

2

27

12

-6

0

(o). In the factorisation $A = LU$ obtained without row exchanges, the matrix L

is upper triangular with ones on its diagonal.

is lower triangular whose sub-diagonal entries are the elimination multipliers.

is always the identity matrix.

is an orthogonal matrix.

carries the pivots on its diagonal.

MCQ Answer Sheet

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(a)	(i)	(ii)	(iii)	(iv)	(v)
(b)	(i)	(ii)	(iii)	(iv)	(v)
(c)	(i)	(ii)	(iii)	(iv)	(v)
(d)	(i)	(ii)	(iii)	(iv)	(v)
(e)	(i)	(ii)	(iii)	(iv)	(v)
(f)	(i)	(ii)	(iii)	(iv)	(v)
(g)	(i)	(ii)	(iii)	(iv)	(v)
(h)	(i)	(ii)	(iii)	(iv)	(v)

(i)	(i)	(ii)	(iii)	(iv)	(v)
(j)	(i)	(ii)	(iii)	(iv)	(v)
(k)	(i)	(ii)	(iii)	(iv)	(v)
(l)	(i)	(ii)	(iii)	(iv)	(v)
(m)	(i)	(ii)	(iii)	(iv)	(v)
(n)	(i)	(ii)	(iii)	(iv)	(v)
(o)	(i)	(ii)	(iii)	(iv)	(v)
