

University of Colombo, Sri Lanka
University of Colombo School of Computing

Bachelor of Science in Computer Science

First Year Examination — Semester I (Sample Paper)

SCS 1306 — Linear Algebra

SAMPLE PAPER A

Two (2) Hours

Answer All Questions

Number of Pages = 16

Number of Questions = 4

To be completed by the candidate

Index Number

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Important Instructions to candidates

- Students should answer in the medium of English language only using the space provided in this question paper.
- Note that questions appear on both sides of the paper. If a page or a part of this question paper is not printed, please inform the supervisor immediately.
- Write your index number **CLEARLY** on each and every page of this Question paper.
- This paper consists 3 structured questions and 15 MCQ based questions on **16** pages.
- Answer **ALL** questions. Questions 1 and 4 carry 30 marks each, while Questions 2 and 3 carry 20 marks each.
- Do not tear off any part of this answer book. Under no circumstances may this book (or any part of this book), used or unused, be removed from the Examination Hall by a candidate.
- Calculators and any electronic device capable of storing and retrieving text including electronic dictionaries, smart watches and mobile phones are **not allowed**.

**To be completed
by the examiners**

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2	
3	
4	
Total	

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1.

- (a). Write down the three elementary row operations used in Gaussian elimination, and state the effect each one has on the determinant of a square matrix.

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- (b). Consider the following linear system.

$$x + 2y + 3z = 9$$

$$2x - y + z = 8$$

$$3x - z = 3$$

- i. Write the augmented matrix of the above linear system.

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- ii. Solve the above linear system using the Gauss–Jordan Elimination Technique and find x , y and z .

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- (c). Let A be the coefficient matrix of a linear system and let $[A \mid \mathbf{b}]$ be its augmented matrix. Explain, in terms of $\text{rank}(A)$, $\text{rank}([A \mid \mathbf{b}])$ and the number of unknowns n , the condition under which the system has (i) no solution, (ii) exactly one solution, and (iii) infinitely many solutions.

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- (d). Consider the following 3×3 matrix A .

$$A = \begin{pmatrix} 2 & 4 & 2 \\ 4 & 9 & 3 \\ 6 & 14 & 8 \end{pmatrix}$$

- i. Find the elimination matrices E_{21} , E_{31} and E_{32} .

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2. Consider the following matrix $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 4 & 1 & 8 \end{pmatrix}$.

(a). Calculate the determinant of A .

(b). Find the cofactor matrix C of A .

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- (c). Determine the adjoint matrix of A .

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- (d). Using the above results, compute the inverse of A .

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- (e). By using a suitable method, justify whether the obtained inverse of A is correct.

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3.

- (a). Define the Singular Value Decomposition (SVD) of a matrix, stating clearly the size and the defining property of each factor.

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- (b). The SVD of a rectangular matrix A is obtained by first forming a symmetric square matrix from A . State the two matrices that can be formed in this way, and explain *one* property they always possess that makes them suitable for this purpose.

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- (c). Consider the following matrix A .

$$A = \begin{pmatrix} 3 & 2 & -2 \\ 2 & 3 & 2 \end{pmatrix}$$

- i. Find the eigenvalues and the corresponding eigenvectors of AA^T .

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- ii. Use the eigenvalues obtained above to compute the singular values of A .

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- iii. Find the decomposed matrices U , Σ and V^T .

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- iv. Verify that the original matrix can be reconstructed correctly from the product of the matrices U , Σ and V^T .

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4. Question 4 has 15 MCQ based questions. (2 marks x 15 = 30 marks)

- Each MCQ question will have 5 (five) choices with one correct answer.
- Mark the correct answers in the provided space at the last page of this paper.
- Cross or color the **best suite answer** among the given options.

(a). If A is a 3×4 matrix and B is a 4×2 matrix, what is the dimension of the product AB ?

4×2

3×2

3×4

2×3

Multiplication not possible

(b). The rank of a matrix is equal to

the total number of rows of the matrix.

the number of non-zero entries of the matrix.

the sum of the diagonal entries of the matrix.

the number of pivots in any echelon form of the matrix.

the largest eigenvalue of the matrix.

(c). For a matrix A of size $m \times n$, the null space $N(A)$ is a subspace of

$\mathbb{R}^m$

$\mathbb{R}^{m+n}$

$\mathbb{R}^n$

$\mathbb{R}^{mn}$

$\mathbb{R}^2$

(d). Which of the following statements is TRUE for all matrices A and B for which the stated operations are defined?

$(AB)^T = B^T A^T$

$(AB)^T = A^T B^T$

$AB = BA$

$\det(A + B) = \det(A) + \det(B)$

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$$(A + B)^2 = A^2 + 2AB + B^2$$

(e). A 5×7 matrix A has rank 4. What is the dimension of the null space of A ?

1

2

4

7

3

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(f). Which one of the following row operations leaves the determinant of a square matrix unchanged?

Interchanging two rows.

Multiplying a row by the scalar 3.

Adding a multiple of one row to a different row.

Replacing a row by a row of zeros.

Dividing a row by its pivot.

(g). A square matrix Q satisfies $Q^T Q = I$. Which of the following is TRUE?

Q is singular.

$\det(Q) = 0$.

Q must be a diagonal matrix.

$Q^{-1} = Q^T$.

Q has no inverse.

(h). For a square matrix A , the sum of all its eigenvalues is equal to

$\det(A)$.

$\text{tr}(A)$.

$\text{rank}(A)$.

the largest pivot of A .

zero, always.

(i). Which of the following statements about the homogeneous system $A\mathbf{x} = \mathbf{0}$ is TRUE?

It never has a solution.

It always has infinitely many solutions.

It is inconsistent whenever A is square.

It has a unique solution only if A is singular.

It always has at least the trivial solution $\mathbf{x} = \mathbf{0}$.

(j). Gauss–Jordan elimination reduces an augmented matrix to

reduced row echelon form.

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lower triangular form.

a diagonal matrix, always.

the identity matrix, always.

an orthogonal matrix.

(k). What is the principal purpose of Principal Component Analysis (PCA)?

To compute the inverse of a covariance matrix.

To convert a matrix into lower triangular form.

To reduce the dimension of a data set while retaining as much of the variance as possible.

To guarantee that a system of linear equations has a unique solution.

To make every rectangular matrix square.

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(l). Which one of the following sets of vectors spans $\mathbb{R}^3$?

$\{(1, 0, 0), (0, 1, 0)\}$

$\{(1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 1)\}$

$\{(1, 1, 1), (2, 2, 2), (3, 3, 3)\}$

$\{(1, 0, 0), (0, 1, 0), (1, 1, 0)\}$

$\{(0, 0, 0)\}$

(m). In the Singular Value Decomposition $A = U\Sigma V^T$, the diagonal entries of Σ are

the eigenvalues of A .

always negative.

the pivots of A .

the non-negative square roots of the eigenvalues of $A^T A$.

the diagonal entries of A .

(n). Given $\mathbf{u} = (1, -2, 3)$ and $\mathbf{v} = (4, 1, -2)$, the value of $\mathbf{u} \cdot \mathbf{v}$ is

−4

4

0

10

−10

(o). A set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\}$ is linearly independent if

at least one $\mathbf{v}_i$ is the zero vector.

every $\mathbf{v}_i$ is a scalar multiple of $\mathbf{v}_1$.

the vectors are mutually orthogonal, and only then.

k is greater than the dimension of the space.

$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \dots + c_k\mathbf{v}_k = \mathbf{0}$ implies $c_1 = c_2 = \dots = c_k = 0$.

MCQ Answer Sheet

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(a)	(i)	(ii)	(iii)	(iv)	(v)
(b)	(i)	(ii)	(iii)	(iv)	(v)
(c)	(i)	(ii)	(iii)	(iv)	(v)
(d)	(i)	(ii)	(iii)	(iv)	(v)
(e)	(i)	(ii)	(iii)	(iv)	(v)
(f)	(i)	(ii)	(iii)	(iv)	(v)
(g)	(i)	(ii)	(iii)	(iv)	(v)
(h)	(i)	(ii)	(iii)	(iv)	(v)

(i)	(i)	(ii)	(iii)	(iv)	(v)
(j)	(i)	(ii)	(iii)	(iv)	(v)
(k)	(i)	(ii)	(iii)	(iv)	(v)
(l)	(i)	(ii)	(iii)	(iv)	(v)
(m)	(i)	(ii)	(iii)	(iv)	(v)
(n)	(i)	(ii)	(iii)	(iv)	(v)
(o)	(i)	(ii)	(iii)	(iv)	(v)
