

University of Colombo, Sri Lanka
University of Colombo School of Computing
BACHELOR OF SCIENCE IN COMPUTER SCIENCE

First Year Examination – Semester I **[SAMPLE PAPER 2]**

SCS 1302 – Discrete Mathematics

(Two (2) Hours)

Answer ALL questions

Number of Pages = 4

Number of Questions = 4

To be completed by the candidate	
Index Number:	

Important Instructions to candidates:

- I. Students should answer in the medium of **English language only** using the answer paper provided.
- II. Note that questions appear on both sides of the paper. If a page or a part of this question paper is not printed, please inform the supervisor immediately.
- III. Write your index number **CLEARLY** on each and every page of the answer paper.
- IV. This paper consists of **4** questions in **4** pages (including the Cover Page).
- V. Answer **ALL** questions. Each question carries **25 marks**.
- VI. Programmable calculators and any electronic device capable of storing and retrieving text, including electronic dictionaries, smart watches and mobile phones, are **not allowed**.
- VII. **Non-programmable** calculators are **not allowed**.
- VIII. Do not tear off any part of this question paper.

To be completed by the examiners

1	
2	
3	
4	
Total	

This is a practice paper. It is modelled on the structure, scope and mark allocation of past SCS 1302 papers.

1.

(a). Which of the following sentences are propositions? For those that are propositions, state the truth value where it can be determined.

- (i). Kandy is a city in Sri Lanka.
- (ii). $x + 4 = 9$.
- (iii). $3 \times 5 = 16$.
- (iv). Close the door.
- (v). Is 17 a prime number?
- (vi). Every even integer greater than 2 is the sum of two primes.

[6 Marks]

(b). Let the domain of discourse be the set of all students at UCSC. Let

$$\begin{aligned}C(x) &: \text{"}x \text{ has taken a course in databases"}, \\L(x) &: \text{"}x \text{ owns a laptop"}, \\F(x, y) &: \text{"}x \text{ and } y \text{ are friends"}.\end{aligned}$$

Express each of the following statements using quantifiers, predicates and logical connectives.

- (i). Every student at UCSC has taken a course in databases.
- (ii). There is a student who owns a laptop but has not taken a course in databases.
- (iii). Every student who owns a laptop has a friend who owns a laptop.
- (iv). No student is a friend of every student.

[8 Marks]

(c). Express the negation of the statement

$$\exists x \forall y (P(x, y) \wedge \neg Q(x, y))$$

so that no negation immediately precedes a quantifier.

[4 Marks]

(d). Using rules of inference, show that the premises

“Every student in this class has a UCSC email account” and “Some student in this class has not activated their account”

imply the conclusion

“Someone who has a UCSC email account has not activated it.”

State the rule of inference used at every step.

[7 Marks]

(Total: 25 marks)

2.

- (a). Let $\mathcal{U} = \{1, 2, 3, \dots, 12\}$ and let

$$A = \{x \in \mathcal{U} : x \text{ is even}\}, \quad B = \{x \in \mathcal{U} : 3 \mid x\}, \quad C = \{x \in \mathcal{U} : x \text{ is prime}\}.$$

Compute each of the following in roster form.

- (i). $A \cap B$
- (ii). $A \cup C$
- (iii). $B - C$
- (iv). $A \oplus B$
- (v). $\overline{A \cup B}$

[10 Marks]

- (b). Let A and B be sets. Prove that

$$A \subseteq B \quad \text{if and only if} \quad A \cup B = B.$$

[7 Marks]

- (c). Let A and B be subsets of a universal set $\mathcal{U}$. Using set identities, show that

$$[(A \cap B) \cup (A \cap \overline{B})] \cup (\overline{A} \cap B) = A \cup B.$$

Name the identity used at every step.

[8 Marks]

(Total: 25 marks)

3.

- (a). Let f be a function from a set A to a set B . Define what it means for f to be

- (i). one-to-one (injective);
- (ii). onto (surjective); and
- (iii). a bijection.

[5 Marks]

- (b). For each of the following functions, determine whether it is one-to-one and whether it is onto. Justify your answer in each case, giving an explicit counterexample or omitted codomain value wherever a property fails.

- (i). $f : \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $f(n) = 3n - 5$.
- (ii). $g : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ defined by $g(n) = \lceil n/2 \rceil$.
- (iii). $h : \mathbb{R} \setminus \{2\} \rightarrow \mathbb{R} \setminus \{1\}$ defined by $h(x) = \frac{x+3}{x-2}$.

[10 Marks]

- (c). Find the inverse of the function h given in part (b)(iii), and verify that $h^{-1}(h(x)) = x$ for all $x \in \mathbb{R} \setminus \{2\}$.

[6 Marks]

- (d). Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Prove that if $g \circ f$ is one-to-one then f is one-to-one. Give an example to show that g need not be one-to-one.

[4 Marks]

(Total: 25 marks)

4.

- (a). Define a *binary relation from a set A to a set B* . Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$. Give an example of a relation from A to B that is *not* the graph of a function from A to B , and state clearly why it is not.

[5 Marks]

- (b). Let R be the relation on the set of integers $\mathbb{Z}$ defined by $a R b$ if and only if $5 \mid (a - b)$.

- (i). Prove that R is reflexive, symmetric and transitive.
(ii). State whether R is antisymmetric, justifying your answer.

[8 Marks]

- (c). Let $A = \{1, 2, 3, 4\}$ and

$$R = \{(1, 2), (2, 2), (2, 3), (3, 1), (3, 4), (4, 4)\}$$

be a relation on A . Compute R^2 and R^3 . Hence determine whether R is transitive, giving a specific ordered pair to justify your conclusion.

[6 Marks]

- (d). Answer the following.

- (i). Find the quotient and the remainder obtained when -137 is divided by 12 , using the division algorithm.

[2 Marks]

- (ii). Use the Euclidean algorithm to find $\gcd(462, 1071)$, showing every division step.

[4 Marks]

(Total: 25 marks)