

University of Colombo, Sri Lanka
University of Colombo School of Computing
BACHELOR OF SCIENCE IN COMPUTER SCIENCE

First Year Examination – Semester I **[SAMPLE PAPER 1]**

SCS 1302 – Discrete Mathematics

(Two (2) Hours)

Answer ALL questions

Number of Pages = 4

Number of Questions = 4

To be completed by the candidate	
Index Number:	

Important Instructions to candidates:

- I. Students should answer in the medium of **English language only** using the answer paper provided.
- II. Note that questions appear on both sides of the paper. If a page or a part of this question paper is not printed, please inform the supervisor immediately.
- III. Write your index number **CLEARLY** on each and every page of the answer paper.
- IV. This paper consists of **4** questions in **4** pages (including the Cover Page).
- V. Answer **ALL** questions. Each question carries **25 marks**.
- VI. Programmable calculators and any electronic device capable of storing and retrieving text, including electronic dictionaries, smart watches and mobile phones, are **not allowed**.
- VII. **Non-programmable** calculators are **not allowed**.
- VIII. Do not tear off any part of this question paper.

To be completed by the examiners

1	
2	
3	
4	
Total	

This is a practice paper. It is modelled on the structure, scope and mark allocation of past SCS 1302 papers.

1.

- (a). Show that the compound propositions

$$(p \rightarrow q) \wedge (\neg p \rightarrow r) \quad \text{and} \quad (p \wedge q) \vee (\neg p \wedge r)$$

are logically equivalent by using a truth table.

[5 Marks]

- (b). Show that the propositions $(p \rightarrow q) \wedge (p \rightarrow \neg q)$ and $\neg p$ are logically equivalent by developing a series of logical equivalences. Name the law used at every step.

[5 Marks]

- (c). In the following program segment, n is an integer variable and A is an array $A[1], A[2], \dots, A[8]$ of 8 integer values.

```
for  $n := 1$  to 8 do  
     $A[n] := 2 * n - 1$ 
```

Translate each of the following statements into logical expressions using quantifiers and logical connectives. Let the domain consist of all the integers from 1 to 8, inclusive.

- (i). Every entry in the array is odd.
- (ii). The entries in the array are in strictly increasing order.
- (iii). No two entries in the array are equal.
- (iv). There exist two entries in A whose sum is 10.

[8 Marks]

- (d). Test the validity of the following argument:

“Every student who submits an assignment late loses marks. No student who loses marks receives a distinction. Nimal received a distinction. Therefore, Nimal did not submit an assignment late.”

Assume the universe is the set of all Computer Science students at UCSC and Nimal is a particular Computer Science student at UCSC.

[7 Marks]

(Total: 25 marks)

2.

- (a). Prove or disprove each of the following statements.

- (i). For every integer n , if $3 \mid n^2$ then $3 \mid n$.

[4 Marks]

- (ii). For every positive integer n , the number $n^2 + n + 41$ is prime.

[4 Marks]

- (iii). For all real numbers x and y , $|x| - |y| \leq |x - y|$.

[4 Marks]

- (b). Let A and B be two subsets of a universal set $\mathcal{U}$. Define the following terms:

- (i). the difference of A and B ($A - B$); and
- (ii). the symmetric difference of A and B ($A \oplus B$).

[3 Marks]

- (c). Let A , B and C be three subsets of a universal set $\mathcal{U}$. Prove or disprove each of the following statements.

(i). $A - (B \cup C) = (A - B) \cap (A - C)$.

[4 Marks]

(ii). $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.

[3 Marks]

(iii). If $A \cap C = B \cap C$, then $A = B$.

[3 Marks]

(Total: 25 marks)

3.

- (a). Consider the binary relation R on the set of integers $\mathbb{Z}$ defined by $(a, b) \in R$ if and only if $a + b$ is even.

Is R reflexive, symmetric, antisymmetric and/or transitive? Justify your answer in each case. State what kind of relation R is.

[8 Marks]

- (b). Let $A = \{2, 3, 4, 6, 8, 12\}$ and let R be the relation on A defined by $a R b$ if and only if $a \mid b$. Show that $(A, \mid)$ is a partially ordered set (poset).

[4 Marks]

- (c). Answer the following questions for the poset $(A, \mid)$ of part (b), where $A = \{2, 3, 4, 6, 8, 12\}$.[†]

- (i). Draw the Hasse diagram for the poset $(A, \mid)$.
- (ii). Find the maximal elements of A .
- (iii). Find the minimal elements of A .
- (iv). Find the greatest element of A , if it exists.
- (v). Find the least element of A , if it exists.
- (vi). Find all upper bounds of $\{2, 3\}$.
- (vii). Find the least upper bound of $\{2, 3\}$, if it exists.
- (viii). Find all lower bounds of $\{8, 12\}$.
- (ix). Find the greatest lower bound of $\{8, 12\}$, if it exists.

[13 Marks]

[†] Part (c) covers maximal/minimal and greatest/least elements and upper/lower bounds of a poset. These appear in past SCS 1302 papers but go beyond the Relations chapter of the accompanying reference note, which stops at Hasse diagrams. The definitions needed are given in the marking scheme.

(Total: 25 marks)

4.

- (a). Let a, b, c, d be integers and let m be a positive integer. Prove that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $ac \equiv bd \pmod{m}$.

[5 Marks]

- (b). Find the least non-negative residue of 2^{500} modulo 13, showing all your working.

[6 Marks]

- (c). State Fermat's Little Theorem, and use it to find the value of $7^{403} \pmod{11}$.

[6 Marks]

- (d). Use the Euclidean algorithm to find $\gcd(120, 253)$. Hence find the inverse of 120 modulo 253, and solve the linear congruence

$$120x \equiv 7 \pmod{253}.$$

[8 Marks]

(Total: 25 marks)